# Finite Math - Fall 2016

Lecture Notes - 11/2/2016

# Section 5.1 - Linear Inequalities in Two Variables

## Application.

**Example 1.** *Define the variable and translate the sentence into an inequality:*

(a) *The number of overtime hours is less than* 20.

(b) *Full-time status requires at least* 12 *credit hours.*

**Solution.**

(a) *Let* $h$ =*number of overtime hours, then* $h < 20$.

(b) *Let* $c$ =*Full-time status, then* $c \geq 12$.

**Example 2.** *Define two variables and translate the sentence into an inequality: Enrollment in finite mathematics plus enrollment in calculus is less than* 300.

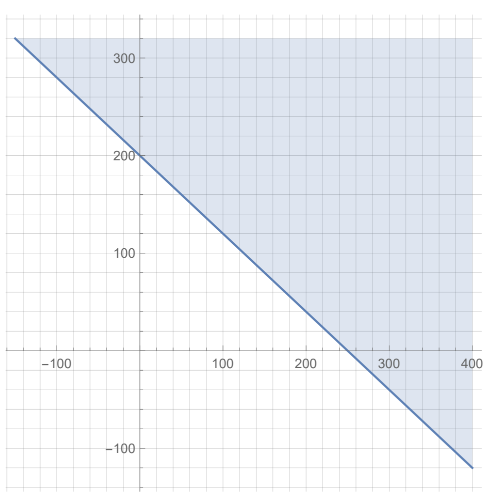
**Solution.** *Let* $F$ *be the enrollment in finite math and let* $C$ *be the enrollment in calculus. Then* $F + C < 300$.

**Example 3.** *A food vendor at a rock concert sells hot dogs for* \$4 *and hamburgers for* \$5*. How many of these sandwiches must be sold to produce sales of at least* \$1,000*? Express the answer as a linear inequality and sketch its graph.*

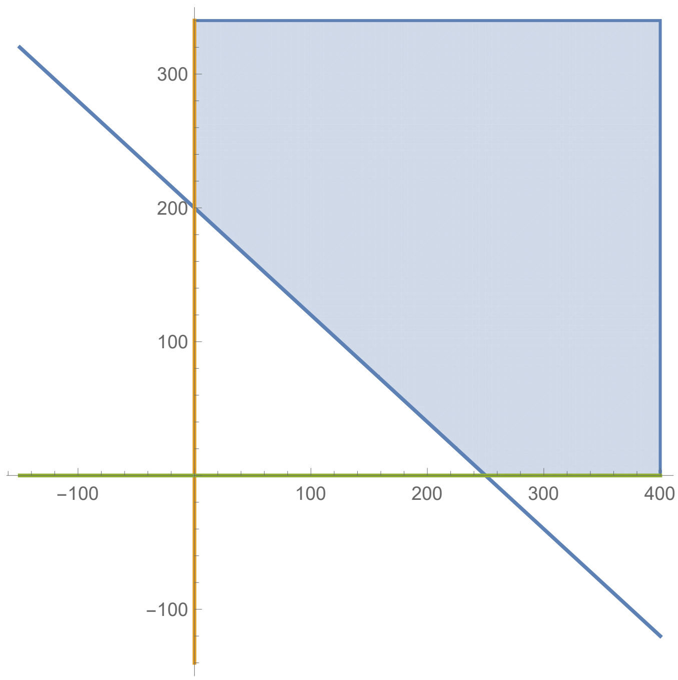
**Solution.** *Suppose the vendor sells* $x$ *hot dogs and* $y$ *hamburgers. Then the seller has made* $4x + 5y$ *dollars. The sellers wants to make at least* \$1000*, so we get*

$$4x + 5y \geq 1000.$$

*If we graph this we get*



*But since a negative number of hot dogs or hamburgers cannot be sold, we also have the inequalities $x \geq 0$ and $y \geq 0$ to add to this which gives the graph*




*The solution is then*

$$\begin{cases} 4x + 5y \geq 1000 \\ x \geq 0, \;\; y \geq 0 \end{cases}$$

**Example 4.** *Seed costs for a farmer are \$40 per acre for corn and \$32 per acre for soybeans. How many acres of each crop should the farmer plant if she wants to spend no more than \$5,000 on seed? Express the answer as a linear inequality and sketch its graph.*

**Example 5.** *A farmer wants to use two brands of fertilizer for his corn crop. Brand A contains 26% nitrogen, 3% phosphate, and 3% potash. Brand B contains 16% nitrogen, 8% phosphate, and 8% potash.*

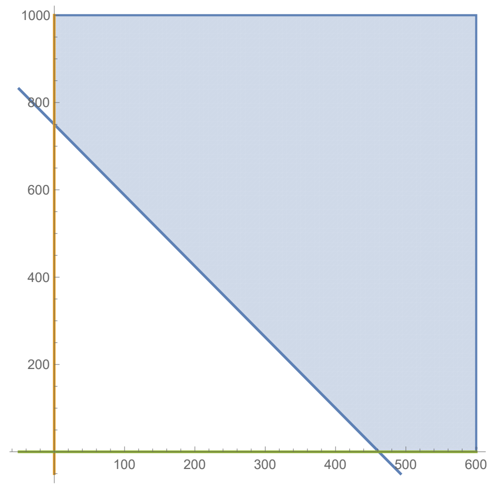
(a) *How many pounds of each brand of fertilizer should he add to each acre if he wants to add at least 120 pounds of nitrogen to each acre?*

(b) *How many pounds of each brand of fertilizer should he add to each acre if he wants to add at most 28 pounds of phosphate to each acre?*

**Solution.**

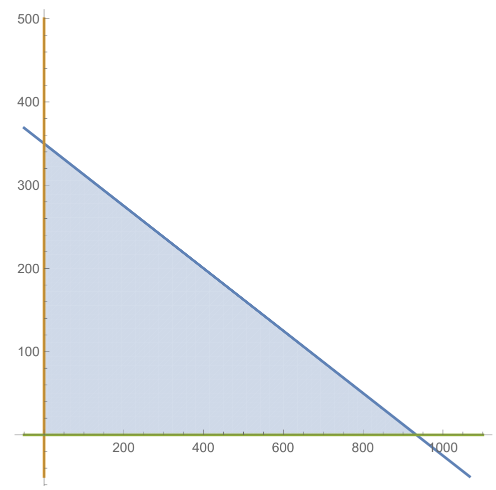
(a) *Let $a$ be the number of pounds of brand A and let $b$ be the number of pounds of brand B. Then*

$$0.26a + 0.16b \geq 120, a \geq 0, b \geq 0$$




(b)

$$0.03a + 0.08b \leq 28, a \geq 0, b \geq 0$$




# Section 5.2 - Systems of Linear Inequalities in Two Variables

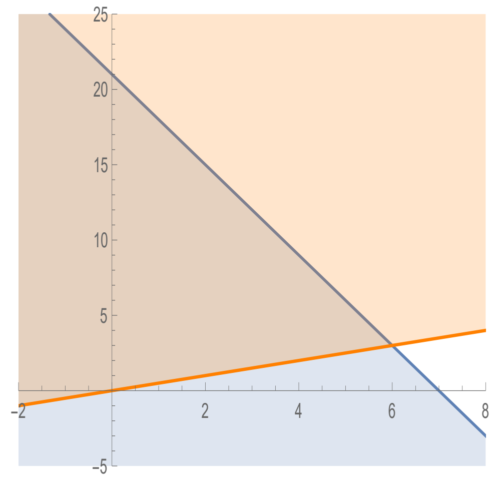
## Solving Systems of Linear Inequalities Graphically.

**Definition 1** (Solution Region/Feasible Region). *Given a system of inequalities, the* solution region *or* feasible region *consists of all points* $(x, y)$ *which simultaneously satisfy all of the inequalities in the system.*

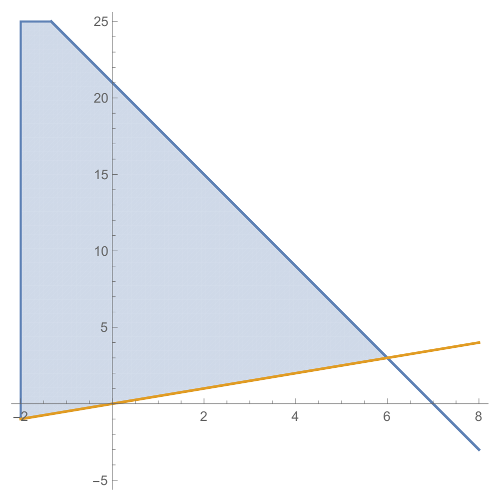
**Example 6.** *Solve the following system of linear inequalities graphically:*

$$\begin{array}{rcrcl} 3x & + & y & \leq & 21 \\ x & - & 2y & \leq & 0 \end{array}$$

**Solution.** *First we begin by graphing both inequalities on the same set of axes*




*then we keep only the portion that the two graphs have in common*




**Example 7.** *Solve the following system of linear inequalities graphically:*

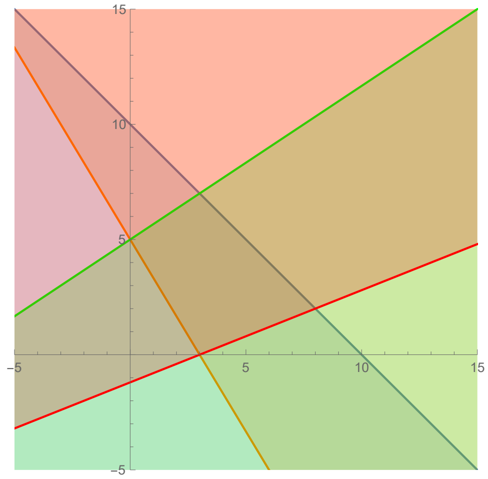
$$\begin{array}{rcrcr} 3x & + & y & \geq & 6 \\ x & - & 5y & \leq & 5 \end{array}$$

**Definition 2** (Corner Point)**.** *A* corner point *of a solution region is a point in the solution region that is the intersection of two boundary lines.*

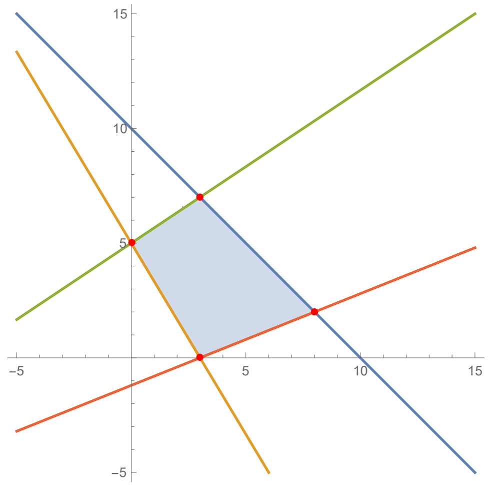
**Example 8.** *Solve the following system of linear inequalities graphically and find the corner points:*

$$\begin{array}{rcrcr} x & + & y & \leq & 10 \\ 5x & + & 3y & \geq & 15 \\ -2x & + & 3y & \leq & 15 \\ 2x & - & 5y & \leq & 6 \end{array}$$

**Solution.** *Begin by plotting all of the inequalities*




*Blue is* $x + y \leq 10$, *orange is* $5x + 3y \geq 15$, *green is* $-2x + 3y \leq 15$, *and red is* $2x - 5y \leq 6$. *Then we keep only the portion that the four graphs have in common*




*In the above graph, the four corner points have been highlighted. To find these, we have to solve the systems of equations each intersection comes from. The intersections come from blue and green, blue and red, orange and green, and orange and red. Using the graphing method AND CHECKING THE SOLUTIONS, we can find that the corner points are*

| *colors* | *system* | *corner point* |
|---|---|---|
| *blue and green* | $\begin{cases} x + y = 10 \\ -2x + 3y = 15 \end{cases}$ | $(3, 7)$ |
| *blue and red* | $\begin{cases} x + y = 10 \\ 2x - 5y = 6 \end{cases}$ | $(8, 2)$ |
| *orange and green* | $\begin{cases} 5x + 3y = 15 \\ -2x + 3y = 15 \end{cases}$ | $(0, 5)$ |
| *orange and red* | $\begin{cases} 5x + 3y = 15 \\ 2x - 5y = 6 \end{cases}$ | $(3, 0)$ |

**Example 9.** *Solve the following system of linear inequalities graphically and find the corner points:*

$$\begin{array}{rcrcl} 5x & + & y & \geq & 20 \\ x & + & y & \geq & 12 \\ x & + & 3y & \geq & 18 \\ & & x & \geq & 0 \\ & & y & \geq & 0 \end{array}$$

**Definition 3** (Bounded/Unbounded)**.** *A solution region of a system of linear inequalities is* bounded *if it can be enclosed within a circle. If it cannot be enclosed within a circle, it is* unbounded*.*

**Remark 1.** *The solution region in Example 6 is unbounded and the solution region in Example 8 is bounded.*

## Applications.

**Example 10.** *A manufacturing plant makes two types of inflatable boats–a two-person boat and a four-person boat. Each two-person boat requires* 0.9 *labor-hour in the cutting department and* 0.8 *labor-hour in the assembly department. Each four-person boat requires* 1.8 *labor-hours in the cutting department and* 1.2 *labor-hours in the assembly department. The maximum labor-hours available each month in the cutting and assembly departments are* 864 *and* 672*, respectively.*

(a) *Summarize this information in a table.*

(b) *If $x$ two-person boats and $y$ four-person boats are manufactured each month, write a system of linear inequalities that reflects the conditions indicated. Graph the feasible region.*

**Solution.**

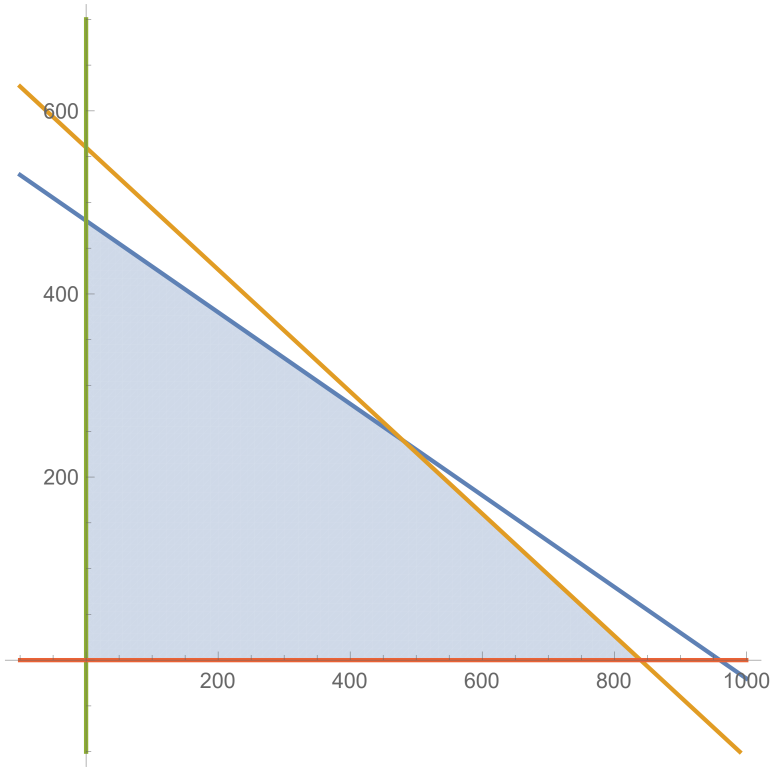
(a) *Begin by organizing the information into a table.*

| | *Two-Person Boat Labor-Hours* | *Four-Person Boat Labor-Hours* | *Maximum Labor-Hours Available per Month* |
|---|---|---|---|
| *Cutting* | 0.9 | 1.8 | 864 |
| *Assembly* | 0.8 | 1.2 | 672 |

(b) *The table lets us quickly come up with the system of inequalities*

$$\begin{array}{rcrcl} 0.9x & + & 1.8y & \leq & 864 \\ 0.8x & + & 1.2y & \leq & 672 \\ & & x & \geq & 0 \\ & & y & \geq & 0 \end{array}$$

*Graphing all of these gives us*




**Example 11.** *A manufacturing company makes two types of water skis, a trick ski and a slalom ski. The trick ski requires* 6 *labor-hours for fabricating and* 1 *labor-hour for finishing. The trick slalom requires* 4 *labor-hours for fabricating and* 1 *labor-hour for finishing. The maximum labor-hours available per day for fabricating and finishing are* 108 *and* 24*, respectively. If* $x$ *is the number of trick skis and* $y$ *is the number of slalom skis produced per day, write a system of linear inequalities that indicates appropriate restraints on* $x$ *and* $y$*. Find the set of feasible solutions graphically for the number of each type of ski that can be produced.*